the solution of Launder et al.[6] using a k-ϵ model of turbulence. It can be seen that the best correlation with measurements is given by Eq. (8) with $\sigma_k = 0.74$, and the worst agreement by the self-similar solution of Eq. (1) without assuming local equilibrium of turbulence. This clearly indicates the influence of the local equilibrium assumption on the k distribution, and points to the fact that such an assumption is quite valid for self-preserving round jets.

Consistent with the local equilibrium assumption, one would expect the turbulence field to be isotropic. This suggests a comparison of Eq. (8) with isotropic dissipation rate data.[3] The comparison (see Fig. 3) is reasonably good qualitatively. However, as expected, Eq. (8) is not in agreement with the semi-isotropic measurements.[3] Because there are no data on ϵ_0, the decay law for ϵ_0 cannot be verified. In spite of this, an indirect verification of $\epsilon_0 \alpha x^{-3}$ can be obtained by considering the expression $\nu_t = C_\mu k^2/\epsilon$ used in the k-ϵ model of turbulence. Launder et al.[6] find it necessary to decrease C_μ with x, otherwise the jet spread will be overpredicted. If $k_0 \alpha x^{-1}$ and $\epsilon_0 \alpha x^{-3}$ are assumed, then $\nu_t = \nu_t(\eta)$ only if $C_\mu \alpha x^{-1}$, which is in qualitative agreement with the suggestion of Launder et al.[6]

Conclusion

It may be said that $\nu_t(\eta)$ prescribed by Eq. (5) gives the best correlation with measurement for k and further substantiates the claim of So and Hwang[1] that the solution [Eqs. (3-7)] represents the best solution for self-preserving, turbulent round jets. Furthermore, Eqs. (3-7) also represent the solutions for incompressible heated round jets. If the turbulent Prandtl number Pr_t is assumed to be constant, then Eq. (3) with the exponent $\eta^2 \ln 2$ replaced by $\eta^2 Pr_t \ln 2$ becomes the solution of the temperature equation. Since Eqs. (1) and (2) are equally applicable for incompressible heated round jets, Eqs. (8-11) also represent solutions for such flows. Therefore, the present results are just as valid for nonisothermal round jets.

Acknowledgment

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Numerical Solution to Rarefaction or Shock Wave/Duct Area-Change Interaction

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Introduction

THERE is a growing interest in the simulation of the flowfields arising from the interaction of rarefaction or shock waves with a segment of the area change in a duct of an elsewhere uniform cross section. We believe that this class of flow problems constitutes a useful model for the analysis of the phenomena encountered in various areas of engineering practice and research, such as piping systems of reciprocating pumps or engines, gas transportation pipelines, shock tubes, and blast wave simulators that incorporate area-change segments in their driver or channel sections.

Early attempts at solving such flowfields[1,2] were hampered by the lack of adequate computers. Only flows in uniform cross sections were computed as truly nonstationary, using the method of characteristics. The flow across each area change was treated as being steady.[1,2] With the advent of modern computers, fully nonstationary solutions became feasible. One of the frequently used numerical schemes is the random choice method (RCM).[3-5] In RCM, which is of first-order accuracy, shock waves and contract discontinuities are sharply defined, unlike schemes employing artificial viscosity (explicit or implicit in the scheme) that generally "smear" discontinuous jumps over several mesh points.

Recently, the diffraction of shock waves or rarefaction waves from an area change in a duct were studied by Greatrix and Gottlieb,[6] Gottlieb and Igra,[7] and Irgra and Gottlieb[8] using the RCM. The results of these computations can exhibit excessive noise in the computed spatial distributions of pressure and velocity that seems to be directly related to the way in which the flowfield in an element containing a shock wave is chosen, with a probability proportional to the length of pre- or aft-shock portion within the element. This noisy distribution was particularly strong in a large area ratio case (e.g., see Figs. 22 and C5 in Ref. 6, Fig. 9 in Ref. 7, and Fig. 9 in Ref. 9). Igra et al.[9] were able to reduce the noise by using finer grid and a more suitable random number generator. However, they were not able to eliminate it altogether (see Figs. 22-25 of Ref. 9).

The purpose of this Note is to demonstrate the advantage gained by using the higher-order numerical scheme based on a generalized Riemann problem (GRP).[10-13] Its running cost was much lower than that of the fine-grid RCM computations, since noise-free and high-resolution results were obtained with far fewer grid points. (Due to the numerical stability limitation on the integration time step, the amount of computation is proporational to the number of grid points squared in both the RCM and GRP schemes.)

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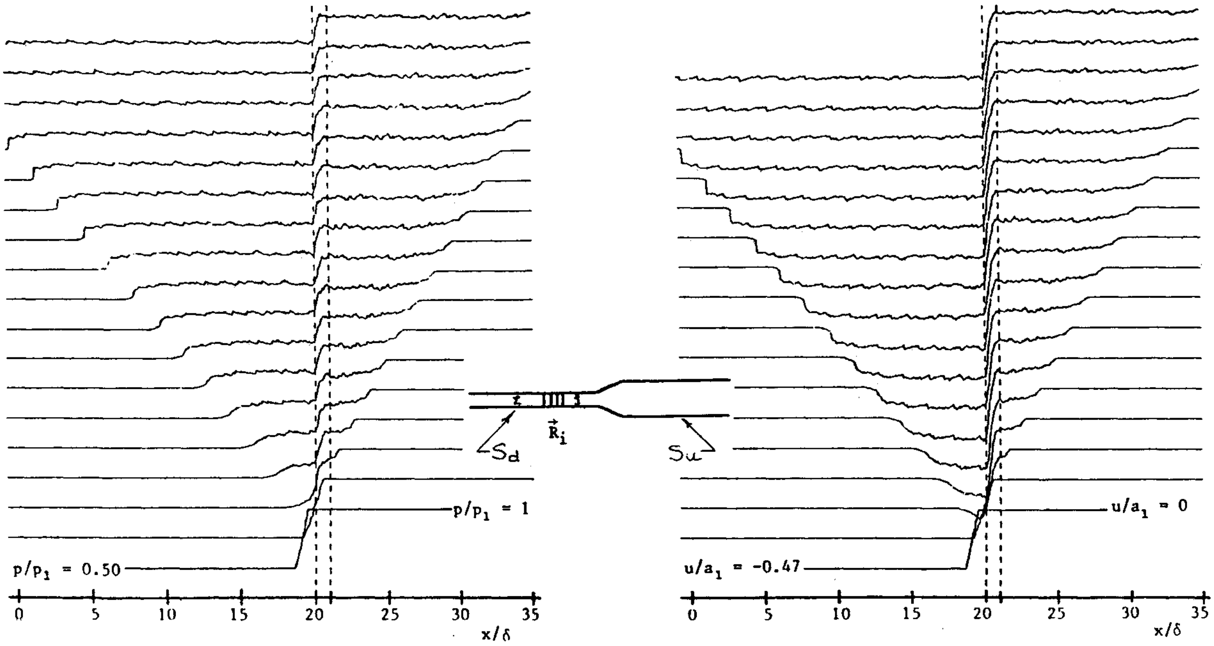

Fig. 1 Spatial distribution of pressure and velocity for the interaction of a rarefaction wave with an area enlargement ($P_2/P_1 = 0.50$, $S_d/S_u = 0.20$) as predicted by the RCM.

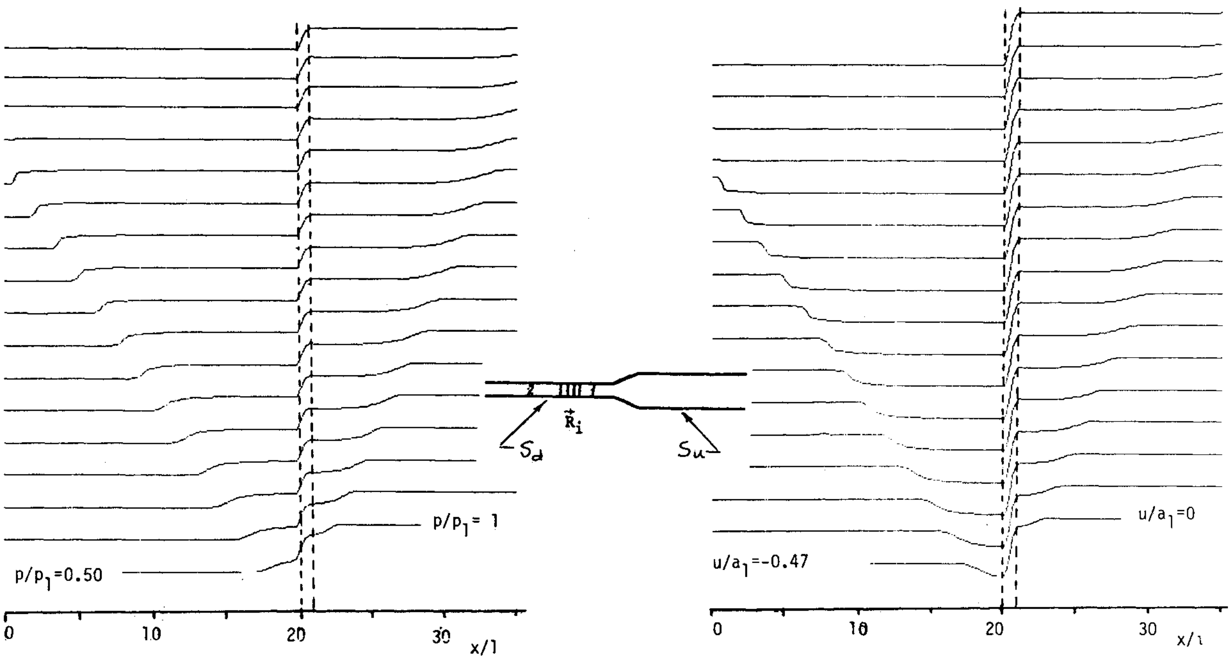

Fig. 2 Spatial distribution of pressure and velocity for the interaction of a rarefaction wave with an area enlargement ($P_2/P_1 = 0.50$, $S_d/S_u = 0.20$) as predicted by the GRP.

The GRP scheme is a second-order extension to the Godunov method, achieved through an analytical solution to a generalized Riemann problem.[10] This scheme was subsequently extended to ducts of a varying cross section[11] and even to a mixed Lagrange/Euler formulation enabling simple and accurate tracing of material discontinuities.[12] A simplified version of the scheme is presented in Ref. 13, which is a good introductory text.

Results and Discussion

As representative samples, three of the cases reported in Refs. 6-9, which contained significant amount of noise, where rerun with the GRP scheme.

In the first case, a centered rarefaction wave of pressure ratio 0.5 that encounters an area enlargement of ratio 0.2 is considered. Figure 1 shows the spatial pressure and velocity distributions as they evolve in time (computed by RCM method[7]). Typically, as the reflected and transmitted waves have swept through a distance much larger than the length of the area-change segment, those "quiet" portions of the flow having a nearly zero gradient become apparent. This is due to the self-similar property of the centered rarefaction wave. The RCM method (Fig. 1) produces a rather noisy distribution for those quiet zones. It may lead to the erroneous conclusion that the quiet zones are not actually obtained as the large-time numerical solution. Solving the same gasdynamic

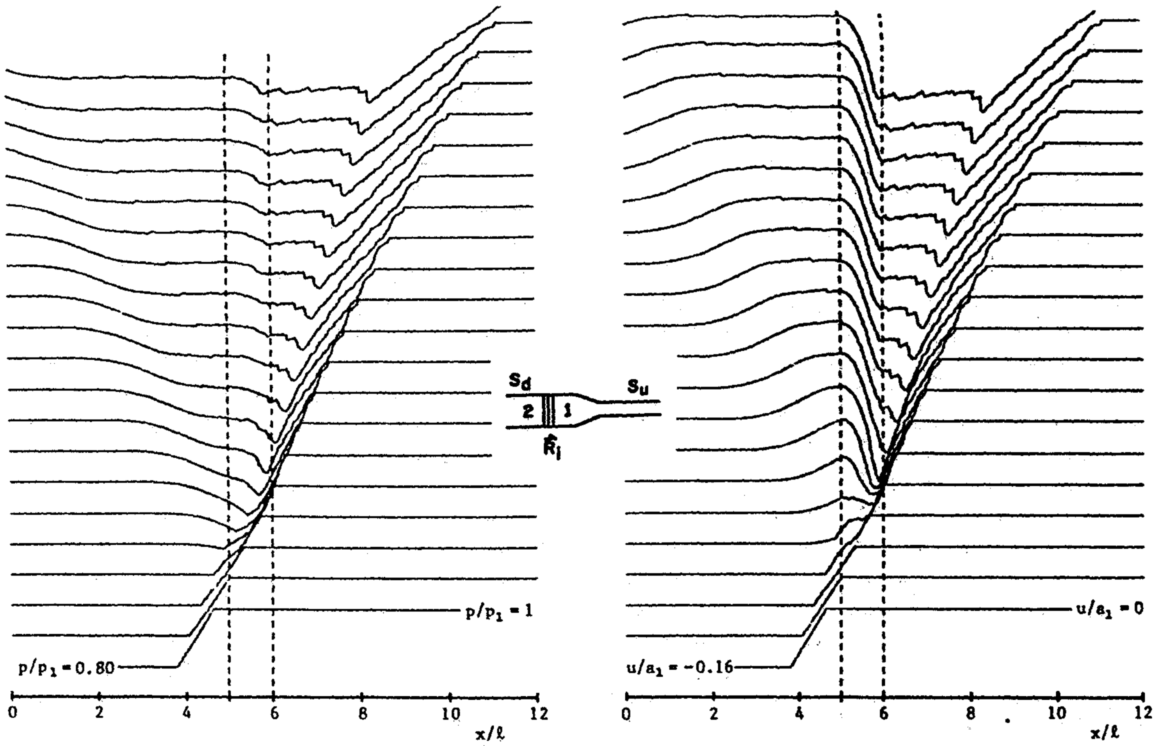

Fig. 3 Spatial distribution of pressure and velocity for the interaction of a rarefaction wave with an area reduction ($P_2/P_1 = 0.80$, $S_u/S_d = 0.25$) as predicted by the RCM.

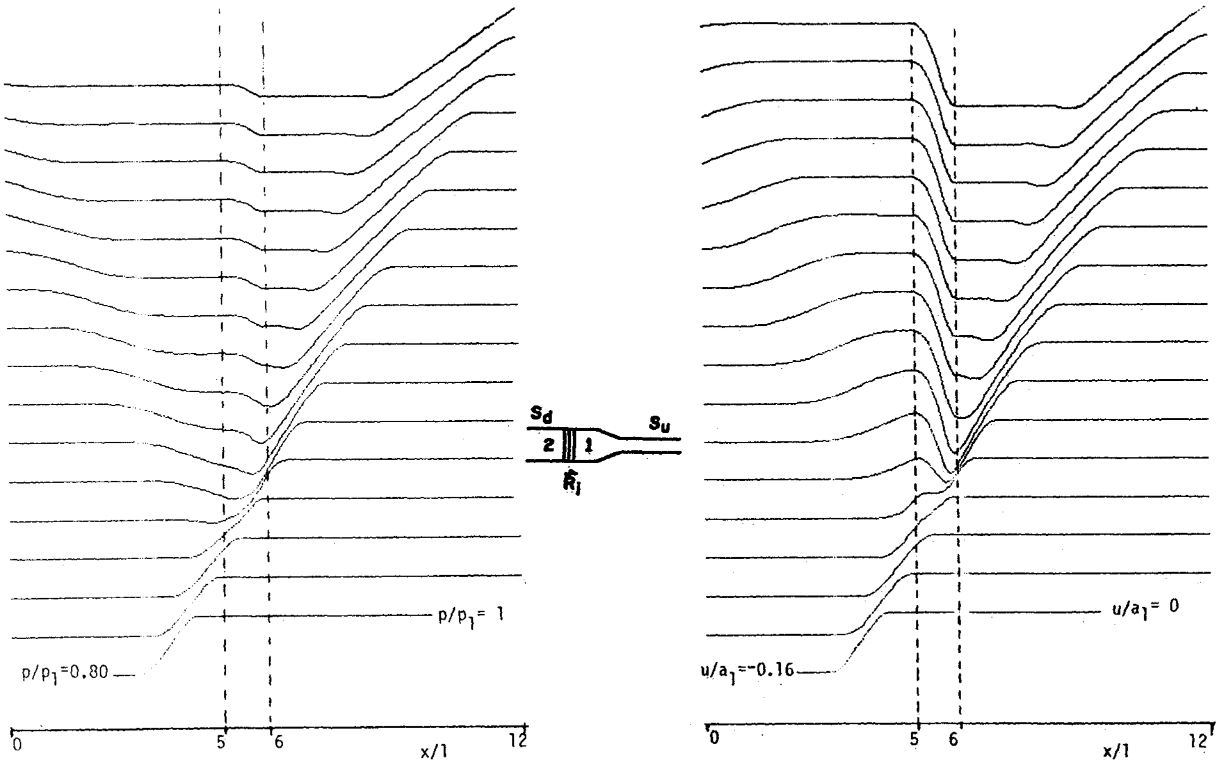

Fig. 4 Spatial distribution of pressure and velocity for the interaction of a rarefaction wave with an area reduction ($P_2/P_1 = 0.80$, $S_u/S_d = 0.25$) as predicted by the GRP.

interaction using the GRP method, as shown in Fig. 2, does indeed exhibit the expected large-time approach to quiet zones upstream and downstream from the area-change segment. Furthermore, while in the RCM computation 720 grid points were used, only 180 points were used in the GRP computation. This represents a reduction of the amount of computations by a factor of about 1/16, since the number of time and space integration steps is proportional to the number of grid points squared. The second case is an interaction of a centered rarefaction (pressure ratio of 0.80) with an area reduction (area ratio 0.25). Again, looking for the quiet zones of the transmitted and reflected waves (Fig. 3), we note a highly noisy RCM solution, particularly on the transmitted side. The GRP solution to the same interaction (Fig. 4) clearly exhibits quiet zones on both sides of the area change, using only 1/6 of the grid points used for the RCM computation (120 vs 720).

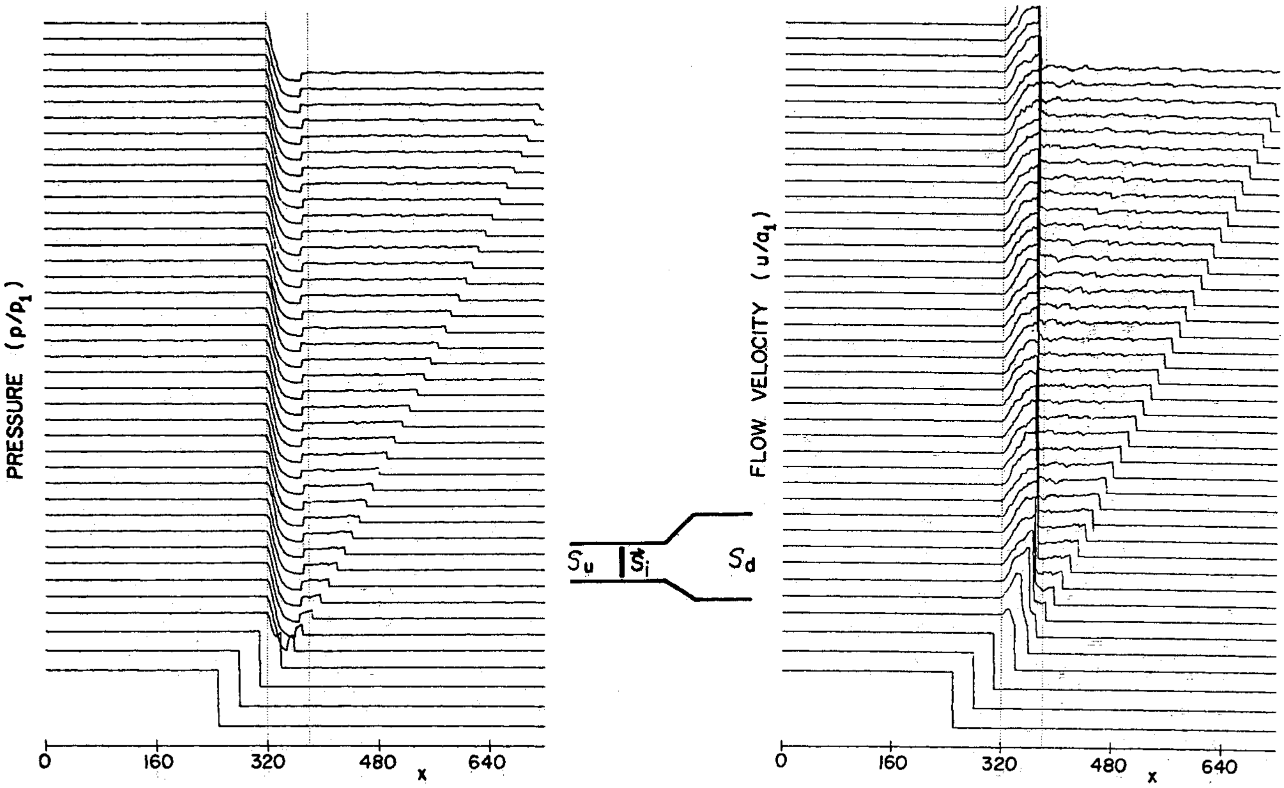

Fig. 5 Spatial distribution of pressure and velocity for the interaction of a shock wave with an area enlargement ($M_s = 4.0$, $S_u/S_d = 0.025$) as predicted by the RCM.

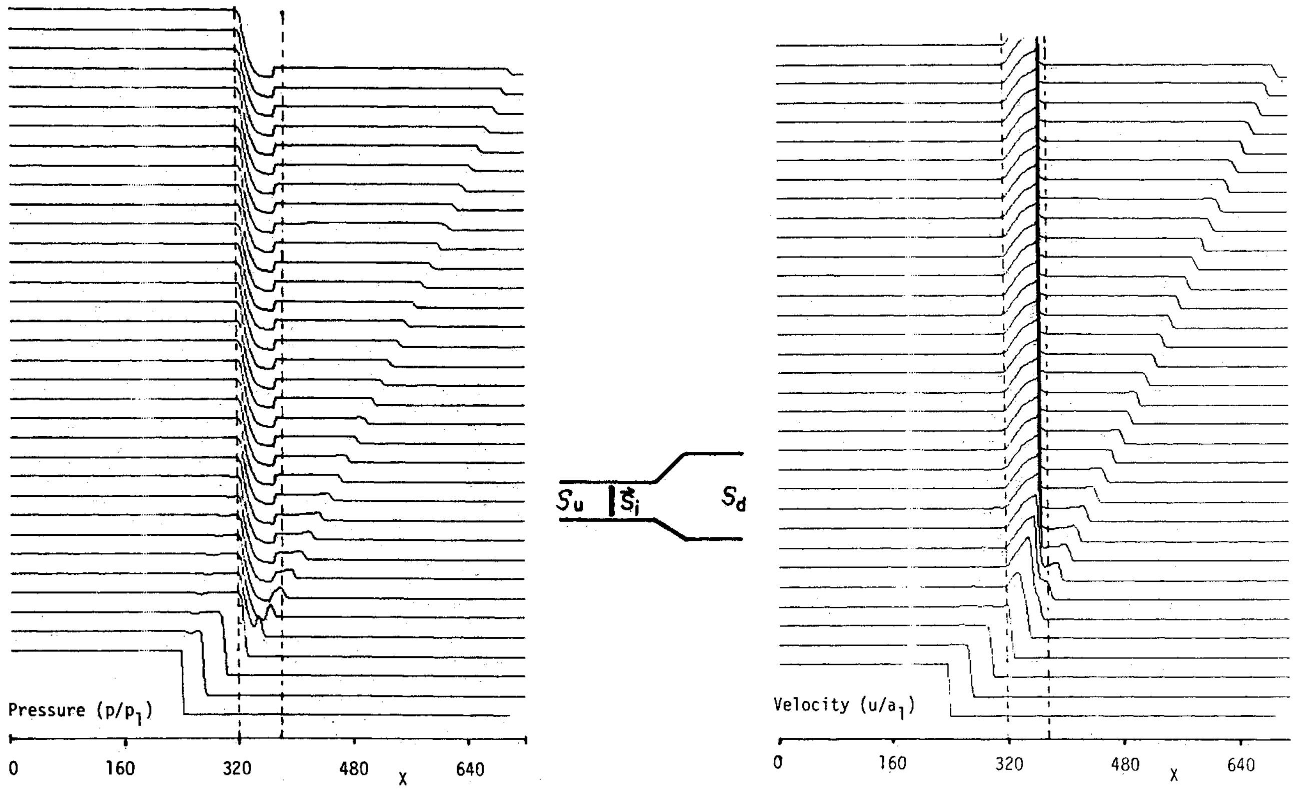

Fig. 6 Spatial distribution of pressure and velocity for the interaction of a shock wave with an area enlargement ($M_s = 4.0$, $S_u/S_d = 0.025$) as predicted by the GRP.

The third case is an interaction of a shock wave ($M=4$) with an area enlargement (area ratio 0.025). The RCM solution (Fig. 5) shows a transmitted shock wave of a reduced strength and a stationary shock wave within the area-change segment. Due to this intermediate shock, there is also a transmitted contact discontinuity moving about midway between the area change and the transmitted shock wave. This contact discontinuity is not shown on the pressure or velocity distributions (it would show up on a density distribution chart). The GRP solution (Fig. 6) shows essentially the same results, except for a remarkably smoother velocity distribution. The RCM grid had 720 points, while the GRP had just 180 points.

Conclusion

The GRP scheme is superior to the RCM for solving the interaction of rarefaction or shock waves interaction with an area change in a duct. It yields practically noise-free results using a much coarser grid than a similar RCM solution.

Acknowledgment

The computations presented here were conducted by a generalized Riemann problem code. The contribution of M. Ben-Artzi to the development of the GRP scheme and code was instrumental and it is gratefully acknowledged.

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Rocket Motor Flow-Turning Losses

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ANALYSES of axial combustion instabilities in solid propellant rocket motors are based on solutions of the gas phase conservation equations in the rocket combustors. A one-dimensional analysis of the unsteady gas motions is the most basic of these formulations. These one-dimensional approaches encounter difficulties in the attempt to account for the multidimensionality of the flow near the burning propellant surface where the gases have both normal and axial velocity components. This problem has been resolved by Culick[1] by the addition of another term, commonly referred to as "flow turning," into the one-dimensional formulation. This flow-turning term is supposed to account for the effect of the turning of the flow from a direction perpendicular to the chamber boundary to the direction of motion of the longitudinal acoustic waves near the burning propellant surface. Analyses by Culick[1] and Flandro[2] show that this process results in energy losses for the combustor wave motion; these losses are referred to as flow-turning losses.

Two interesting questions, addressed most recently by Hersh and Walter,[3-5] arise in connection with the flow-turning losses. The first concerns the adequacy of a one-dimensional analysis, in which the mass injection from the side walls appears as the wave forcing function, to model the flow-turning losses. Since flow-turning losses involve a directional transfer of momentum, it appears that at least a two-dimensional model of the flowfield would be required, as indicated by Flandro.[2] The second question is whether these losses occur in the vicinity of the chamber walls or in the core flow. If the former is true, then it should be possible to describe the wave motion in the bulk of the chamber by a one-dimensional analysis, provided the flow-turning losses are properly accounted for. In the latter case, however, at least a two-dimensional analysis must be carried out over the entire chamber.

To clarify these statements, consider a constant-area rigid walled duct of high length-to-diameter ratio. Lateral injection of flow is therefore absent. Even in such a case, the longitudinal wave motions are not strictly one-dimensional due to viscous and thermal conduction effects near the wall. However, these effects are important only in a thin layer next to the wall known as the acoustic boundary layer, or Stokes' layer. The extent δ of this boundary layer is given by[6]

$$\delta=\sqrt{2\nu/\omega} \tag{1}$$

where ν is the kinematic viscosity and ω the frequency. (The thermal and viscous boundary layers have approximately the same extent for gases whose Prandtl number is close to unity.) A one-dimensional analysis of the acoustic wave motion in the duct requires the conservation equations to be averaged across the cross section. It may be shown that the relevant wave number k_∞ for longitudinal motions is given by

$$k_\infty^2=\left(\frac{\omega}{c}\right)^2-i2\frac{\omega}{c}\frac{\beta}{a}$$

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